E C O N O M I C S B U L L E T I N

Illegal migration and economic growth: simulation analysis in an international context

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Abstract

This paper calibrates the dynamic illegal migration model studied by Hazari and Sgro (2003) to eight countries. We find that if illegal migrants and domestic labor are perfect substitutes in production, the presence of illegal migrants lowers domestic welfare between 0.09% and 2.93% among eight countries. Moreover, we show that there is a tradeoff between the long-run domestic wage rate and rental rate of capital from receiving illegal migrants.

I would like to thank Fred Kwan, Bharat Hazari, Charles Leung, and the associate editor for helpful comments. All errors, of course, are mine.
Citation: Ho, Cheuk Yin, (2007) "Illegal migration and economic growth: simulation analysis in an international context." *Economics Bulletin*, Vol. 6, No. 41 pp. 1-13
Submitted: September 10, 2007. **Accepted:** October 17, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume6/EB-07F20008A.pdf

1 Introduction

In recent years there has been increasing interest in the welfare effect of illegal migration in a dynamic context. This is reflected in the recent writings of Hazari and Sgro (2003) and Moy and Yip (2006). Hazari and Sgro (2003) develop a Ramsey-type dynamic optimizing model to show that illegal migration necessarily lowers the long-run per capita domestic consumption given that illegal migrants and domestic residents are perfect substitutes in production. Moy and Yip (2006) reexamine the Hazari-Sgro model and find that the welfare effect of illegal migration is ambiguous but the effect is positive in the case of Cobb-Douglas production technology.

The purpose of this paper is to quantitatively evaluate the welfare effect of the illegal migration model studied by Hazari and Sgro (2003) and to investigate the extent to which different countries are affected by illegal migration in an international context. Contrary to the analytical result in Hazari and Sgro (2003) that illegal migration *necessarily* lowers the long-run per capita domestic consumption, we numerically show that illegal migration shifts up the time path of per capita domestic consumption, and hence the domestic welfare (aggregate utility) increases between 0.09% and 2.93% among eight countries. From the perspective of the domestic welfare, the U.S. benefits the most and Japan the least from illegal migration. A higher proportion of illegal migrants of total labor force and a higher intensity of wage exploitation on illegal migrants are two major underlying factors that an economy benefits more from illegal migration. In addition, we find that there is a long-run negative relationship between the change in domestic wage rate and the change in rental rate of capital. Therefore, despite an increase in the aggregate welfare of domestic residents, illegal migration could bring about an income distribution problem between the capitalists and domestic workers.

The remainder of this paper is organized as follows. Section 2 reviews the Hazari-Sgro Model. Section 3 calibrates the model and discusses the results. Section 4 concludes with a brief summary.

2 Review of the Hazari-Sgro Model

The economy produces output, *Y*, with the help of capital, *K*, domestic labor, *L*, and illegal migrants, *M*. The production function exhibits (1) constant returns to scale, (2) diminishing returns to factors, and satisfies (3) the Inada conditions. Given that *L* and *M* are perfect substitutes, the production function is given below:

$$Y = F\left(K, L + M\right) \tag{1}$$

We can rewrite the production function in intensive form as:

$$y = \frac{Y}{L+M} = f\left(\frac{K}{L+M}\right) = f(k) \tag{2}$$

where *y* is per capita output and k is per capita capital.

Profit maximization implies that:

$$r = f'(k)$$

$$w = f(k) - f'(k)k$$

$$w_m = \beta w, \quad 0 < \beta < 1$$

where *r* denotes rental rate of capital, *w* wage rate of domestic labor, w_m wage rate of illegal migrants, β intensity of wage exploitation on illegal migrants. The term $0 < \beta < 1$ means that illegal migrants are paid below domestic labor for the same marginal productivity. Domestic consumption can be defined as:

$$C = Y - \dot{K} - \beta w M \tag{3}$$

where C is domestic consumption, $\dot{K}$ investment in capital, βwM wage payment to illegal migrants. It is assumed that illegal migrants do not accumulate capital in the host country and grow at the same rate as domestic labor, n. If we write equation (3) in an intensive form and define $m = M/(L+M)$, $c = C/L$, $k = K/(L+M)$, we can derive the economy-wide resource constraint as:

$$\dot{k} = f(k) - nk - (1-m)c - \beta wm \tag{4}$$

In order to calibrate the model in later section, we reformulate the dynamic optimization problem with CRRA utility function and Cobb-Douglas production function, $y = k^{\phi}$, so the social planner problem is:

$$\max_{c} \int_{0}^{\infty} \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} dt \quad \text{s.t.} \quad \dot{k} = k^{\phi} - nk - (1-m)c - \beta m k^{\phi}(1-\phi)$$

$$k(0) = k_0 \quad \text{and} \quad 0 \le c \le f(k)$$

By applying the Pontryagin's Maximum Principle, this dynamic optimization problem can be solved by maximizing the following Hamiltonian:

$$H = \frac{c^{1-\theta}-1}{1-\theta} e^{-\rho t} + \lambda \left\{ k^{\phi} - nk - (1-m)c - \beta m k^{\phi}(1-\phi) \right\}$$

Solving this dynamic optimization problem and doing some manipulations, we obtain two ordinary differential equations:

$$\dot{k} = k^{\phi} - nk - (1-m)c - \beta m k^{\phi}(1-\phi) \tag{5}$$

$$\dot{c} = \frac{c}{\theta} \left\{ \phi k^{\phi-1} - \beta m \phi (1-\phi) k^{\phi-1} - (n+\rho) \right\} \tag{6}$$

and the following transversality condition:

$$\lim_{t\to\infty}\left[\lambda(t)k(t)\right]=0 \tag{7}$$

3 Model Calibration

3.1 Parameterization

The Hazari-Sgro model is calibrated to eight countries, i.e. the U.S., Canada, Germany, France, Spain, Japan, Sweden, and the U.K., respectively. The parameter values of eight countries are given in Table 1 and the data sources are described in Data Appendix. ϕ is capital share, n is population growth rate, $m=M/(L+M)$ is proportion of illegal migrants of total labor force, and β is intensity of wage exploitation on illegal migrants (or ratio of average wage of illegal migrants to that of domestic labor). Since there are no available data on the wage rate of illegal migrants, so we use the minimum wage rate adopted by each country as a proxy for the average wage rate of illegal migrants in that particular country. This choice is reasonable in that illegal migrants are exploited by domestic employers so that their average wage rate is closely related to the minimum wage rate in the host country. Suppose the government raises the minimum wage rate, then the domestic employers have more incentives to hire illegal migrants to save labor cost, so an increase in labor demand for illegal migrants drives up the average wage rate of illegal migrants. ρ is consumer's subjective discount rate and θ is elasticity of the marginal utility of consumption. Following the economic growth literature, we calibrate ρ to 0.02 and θ to 2.

3.2 Domestic Welfare

In order to accurately measure the impact of illegal migration on the domestic welfare, we do not only look at per capita domestic consumption in the steady state but also in the transitional dynamics. So, we numerically solve out the time path of per capita

domestic consumption. Figure 1 shows the case of the U.S. with the time path of per capita domestic consumption shifts up in the presence of illegal migration. This means that illegal migration increases per capita domestic consumption in both the transitional dynamics and the steady state. Therefore, our numerical result is in contrast to the analytical result of Hazari and Sgro (2003) because they argue that illegal migration *necessarily* lowers the long-run per capita domestic consumption for all production functions. Also, our numerical result support the analytical finding of Moy and Yip (2006) that the long-run effect on per capita domestic consumption is positive in the case of Cobb-Douglas production function.

Since we have already found out the time path of per capita domestic consumption, we easily obtain the domestic aggregate utility by numerically integrating the discounted value of per capita domestic consumption in the following intertemporal utility function:

$$U = \int_0^{\infty} \frac{c(t)^{1-\theta} - 1}{1-\theta} e^{-pt} dt$$

It is shown in Table 2 that illegal migration raises the domestic welfare between 0.09% and 2.93% among eight countries. For example, the U.S. has the largest percentage increase in the domestic welfare (2.93%), Japan the lowest (0.09%), and other European countries between 0.31% and 2.17%. The percentage increase in the domestic welfare of the U.S. is about 33 times of Japan's. The intuition behind this result is straightforward. Given that illegal migrants do not save in domestic economy, an inflow of illegal migrants lowers the accumulation of per capita capital in domestic economy that generates a negative intertemporal effect. On the other hand, domestic economy employs cheap illegal workers and then reaps some profits, which generates a positive "wage exploitation" effect. After all, the increase in per capita domestic consumption implies that the positive "wage exploitation" effect strictly dominates the negative intertemporal effect in the case of Cobb-Douglas production technology.

We proceed to analyze why different countries have different increases in domestic

welfare under illegal migration. As shown in Table 2 we observe that countries with a higher increase in domestic welfare, namely, the U.S. and Spain, have a higher value of m, which means that they have a higher proportion of illegal migrants of total labor force. In addition, both of them have a lower value of β, which means that the average wage rate of illegal migrants is substantially lower than that of domestic labor. Therefore, this implies that a larger population of low-paid illegal migrants generates a higher economic profit for domestic economy and thus a higher increase in domestic welfare. On the other hand, even though the average wage rate of illegal migrants relative to domestic labor in Japan is low ($\beta = 0.33$), her proportion of illegal migrants of total labor force is also very low ($m = 0.003$), so Japan cannot gain very much from "wage exploitation" on illegal migrants.

3.3 Domestic Wage Rate and Rental Rate of Capital

Table 2 shows that the U.S. has the largest decrease in steady-state domestic wage rate (-0.70%), followed by Spain (-0.39%), and Japan the least (-0.04%). The percentage decline in domestic wage rate of the U.S. is about 18 times of Japan's. The decline in domestic wage rate can be explained by the reason that illegal migrants are perfect substitutes for domestic labor in production, so the presence of illegal migrants increases the labor supply in domestic economy that lowers the marginal productivity and thus the wage rate of domestic labor.

On the other hand, Table 2 shows that illegal migration raises the long-run rental rate of capital and the multitude is between 0.003% and 0.011% among eight countries. In other words, there is a negative long-run relationship between the change in domestic wage rate and the change in rental rate of capital as depicted in Figure 2. From receiving illegal migrants, the economy has to face a tradeoff between a lower domestic wage rate and a higher rental rate of capital. Therefore, illegal migration raises the aggregate welfare but also could bring about an income distribution problem between the capitalists and domestic workers in the economy. This explains why there are illegal migrants in the economy and the capitalists are willing to employ illegal migrants but the labor unions

always fight against the employment of them. Since in a representative agent model the welfare between the capitalists and domestic workers cannot be separately measured, so a deeper understanding about the income distribution issue on the welfare of the capitalists and domestic workers can be further investigated in some heterogeneous agent models. In addition, the Hazari-Sgro model also generates a migration policy implication that there could be an optimal number of illegal migrants if the government does not only maximize the aggregate welfare but also considers the income distribution problem in the economy.

4 Concluding Remarks

By calibrating the Hazari-Sgro Model we have found that illegal migration does not necessarily lower but instead raises the domestic welfare when illegal migrants are perfect substitutes for domestic labor and that the increase in domestic welfare gained by different countries varies considerably. According to our calibration results, the U.S. benefits the most and Japan the least. Last, our calibration results shed some light on the issue of income distribution between the capitalists and domestic workers as there is a long-run negative relationship between the change in domestic wage rate and the change in rental rate of capital under illegal migration.

Figure 1: The time path of per capita domestic consumption of the U.S.

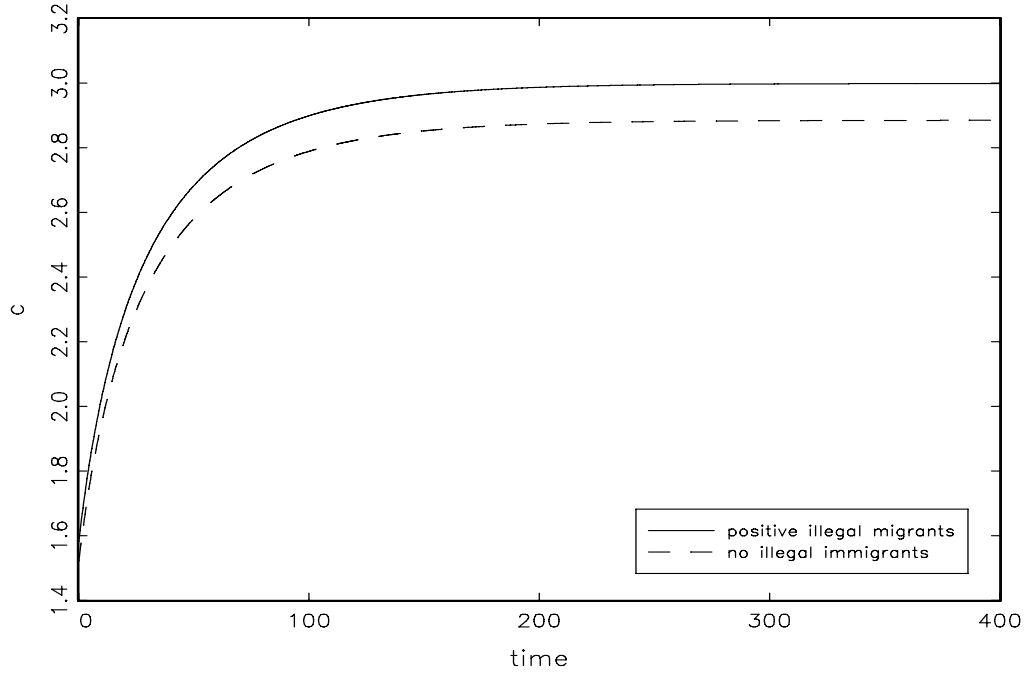

Figure 2: The relationship between the percentage change in steady-state domestic wage rate and the percentage change in steady-state rental rate of capital.

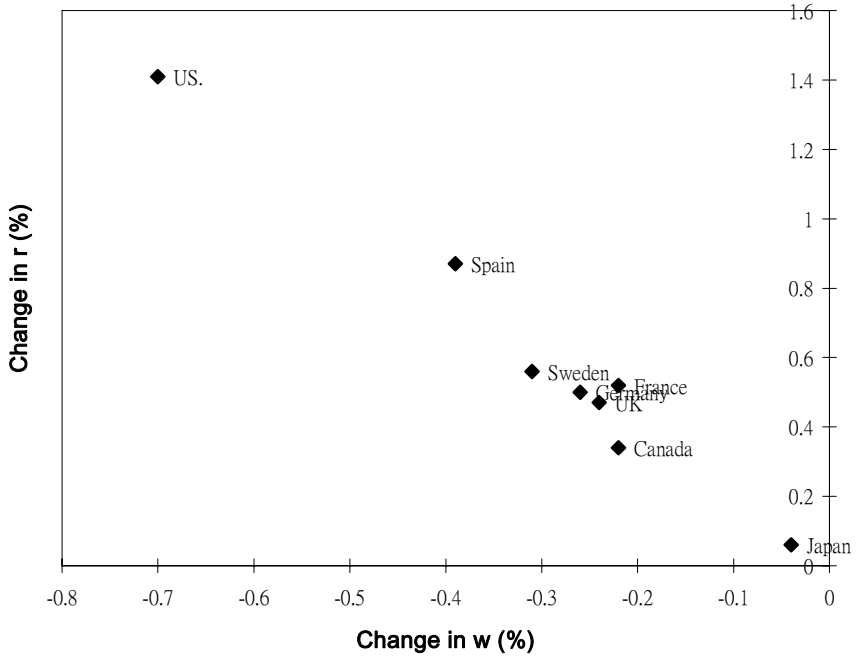

Table 1: Parameter values for eight countries.

Country	ϕ	β	m	n
U.S.	0.333	0.36	0.058	0.0110
Canada	0.400	0.43	0.013	0.0110
Germany	0.340	0.58	0.013	0.0029
France	0.299	0.62	0.012	0.0047
Spain	0.310	0.32	0.039	0.0055
Japan	0.362	0.33	0.003	0.0031
Sweden	0.355	0.51	0.017	0.0038
U.K.	0.340	0.42	0.017	0.0029

Table 2: The percentage changes in domestic welfare, steady-state domestic wage rate, and steady-state rental rate of capital for eight countries.

Country	Change in domestic welfare (%)	Change in steady-state domestic wage rate (%)	Change in steady-state rental rate of capital (%)
U.S.	2.93	-0.70	1.41
Canada	0.31	-0.22	0.34
Germany	0.31	-0.26	0.50
France	0.40	-0.22	0.52
Spain	2.17	-0.39	0.87
Japan	0.09	-0.04	0.06
Sweden	0.43	-0.31	0.56
U.K.	0.56	-0.24	0.47

Data Appendix

Table 3: Parameter descriptions and sources

Parameter	Description	Source
n	Average of population growth rate between 1985 and 2004	World Development Indicators (2006)
L	Average of domestic labor force between 1985 and 2004	World Development Indicators (2006)
M	Current population of illegal migrants	Table 2
m	$M / L + M$	Author's calculations
ϕ	Physical income share	Table 2
β	Ratio of minimum wage rate to average wage rate	Neumark and Wascher (2003)

Table 4: Parameter sources for eight countries

Country	ϕ	M
U.S.	King and Rebelo (1999)	http://pewhispanic.org/
Canada	Murchison et al. (2004)	http://are.berkeley.edu/APMP/pubs/agworkvisa/canada111503.html
Germany	Ertz (1997)	http://www.fnstusa.org/ImmigrationGermany.pdf
France	Conesa and Kehoe (2005)	http://www.washtimes.com/upi-breaking/20050511-100133-3494r.htm
Spain	Conesa and Kehoe (2005)	http://www.ipsnews.net/news.asp?idnews=33064
Japan	Hayashi and Prescott (2002)	http://www.jil.go.jp/bulletin/year/2003/vol42-06.pdf
Sweden	Linde (2004)	http://www.weeklystandard.com/Content/Public/Articles/000/000/005/271dgkju.asp
U.K.	Kapetanios et al. (2007)	http://www.timesonline.co.uk/article/0,,2087-1572533,00.html

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